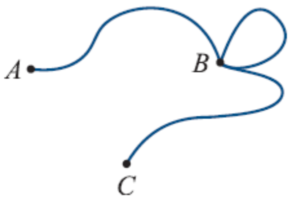
Matrix:	A matrix is a rectangular array of numbers. Matrices are normally labelled with a capital letter Example: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
Order of a matrix:	(Rows x Columns) Example: $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}\begin{matrix} row\ 1 \\ row\ 2 \end{matrix}$ $column\ 1 \quad column\ 2 \quad column\ 3$ Order: (2 x 3)
Elements of a matrix:	• The numbers inside a matrix. Position of an element identified by a lower case letter and two subscript numbers. • Letter refers to a particular matrix, first number refers to row reference, second number refers to column reference. Example: $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$ $C = \begin{bmatrix} 9 \\ 10 \\ 11 \end{bmatrix}$ $a_{13} = 3$ $b_{21} = 8$ $c_{31} = 11$ Ie: a_{13} = element in matrix A, position row 1 column 3
Network:	• The numbers in the matrix represent the edges leading from one vertex to another, directly.  $\begin{matrix} & A & B & C \\ A & 0 & 1 & 0 \\ = B & 1 & 1 & 1 \\ C & 0 & 1 & 0 \end{matrix}$ Loops have a degree of 2 but are only considered as 1 edge • Rows represent 'From' Columns represent 'To' • Sum of each row represents total paths ***from*** each particular vertex • Sum of each column represents total paths ***to*** each particular vertex
Adding/subtracting matrices:	• Two matrices can only be added and subtracted if they have the same order. • Add or subtract the elements in the corresponding positions $A = \begin{bmatrix} 10 & 5 \\ 8 & -1 \end{bmatrix} B = \begin{bmatrix} 0 & -6 \\ 2 & 3 \end{bmatrix} C = \begin{bmatrix} 4 & 12 \\ 9 & -10 \end{bmatrix}$ Example 1: $A + B = \begin{bmatrix} 10+0 & 5+-6 \\ 8+2 & -1+3 \end{bmatrix} = \begin{bmatrix} 10 & -1 \\ 10 & 2 \end{bmatrix}$ Example 2: $C - A = \begin{bmatrix} 4-10 & 12-5 \\ 9-8 & -10--1 \end{bmatrix} = \begin{bmatrix} -6 & 7 \\ 1 & -9 \end{bmatrix}$
Zero matrix:	• All elements are zeros • Typically labelled as 0 • Can be any a matrix of any order $0 = [0]$ $0 = [0 \quad 0 \quad 0]$ $0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Identity matrix:	• Leading diagonal elements are one all other elements are zeros • Typically labelled as I • Can be a **square** matrix of any size *(square matrix: rows = columns)* $I = [1] \qquad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Scalar multiplication:	• Multiply all elements in a matrix by a specified coefficient. $A = \begin{bmatrix} 10 & 5 \\ 8 & -1 \end{bmatrix} B = \begin{bmatrix} 0 & -6 \\ 2 & 3 \end{bmatrix}$ Example 1: $2A = 2\begin{bmatrix} 10 & 5 \\ 8 & -1 \end{bmatrix} = \begin{bmatrix} 2 \times 10 & 2 \times 5 \\ 2 \times 8 & 2 \times -1 \end{bmatrix} = \begin{bmatrix} 20 & 10 \\ 16 & -2 \end{bmatrix}$ Example 2: $-5B = -5\begin{bmatrix} 0 & -6 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} -5 \times 0 & -5 \times -6 \\ -5 \times 2 & -5 \times 3 \end{bmatrix} = \begin{bmatrix} 0 & 30 \\ -10 & -15 \end{bmatrix}$
Matrix multiplication:	• Two matrices can only be multiplied if the number of columns in the 1st matrix is equal to the number of rows in the 2nd matrix. • If the two orders of both matrices are placed together, the inside numbers identify whether the two matrices can be multiplied (same = can be multiplied) and the outside numbers identify the **resultant matrix** (the order of the product) Example 1: $A = [a \quad b] B = \begin{bmatrix} c & d \\ e & f \end{bmatrix}$ $(1 \times 2)(2 \times 2)$ Resultant matrix: (1×2) Number are the same: can be multiplied $AB = [(ac + be) \quad (ad + bf)]$ Example 2: $C = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} D = \begin{bmatrix} m & r \\ s & t \\ u & x \end{bmatrix}$ $(2 \times 3)(3 \times 2)$ Resultant matrix: (2×2) Number are the same: can be multiplied $CD = \begin{bmatrix} (am + bs + cu) & (ar + bt + cx) \\ (dm + es + fu) & (dr + et + fx) \end{bmatrix}$
Determinant: Denoted as: Δ, detA or $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$	• As long as the determinant ***does not*** equal to zero, an inverse can be found and a unique solution to simultaneous equations exists. $\Delta = ad - bc \qquad where\text{:} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ Example: $A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$ $\Delta A = (2 \times 1) - (3 \times 4) \quad \Delta A = -9$

Inverse: Denoted as: A^{-1}	$$A^{-1} = \frac{1}{determinant}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$ • To find the inverse of a matrix, where the determinant ≠ 0 1. Swap elements *a* and *d* 2. Change the signs of elements *b* and *c* 3. Divide each number by the value of the determinant Example: $$A = \begin{bmatrix} 3 & 0 \\ -6 & 1 \end{bmatrix}$$ $$\Delta A = (3 \times 1) - (0 \times -6) \quad \Delta A = 3$$ $$A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 0 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} 1/3 & 0 \\ 2 & 1 \end{bmatrix}$$
Solving simultaneous equations:	$$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1} \times B$$ • Represent equations in matrix form. • Coefficient matrix = A • Answer matrix = B Example: $$4x - 3y = 10$$ $$3x + y = 1$$ $$\begin{bmatrix} 4 & -3 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 1 \end{bmatrix}$$ $$\Delta A = (4 \times 1) - (-3 \times 3) \quad \Delta A = 13$$ $$A^{-1} = \frac{1}{13}\begin{bmatrix} 1 & 3 \\ -3 & 4 \end{bmatrix}$$ $$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{13}\begin{bmatrix} 1 & 3 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 10 \\ 1 \end{bmatrix}$$ $$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{13}\begin{bmatrix} (1 \times 10 + 3 \times 1) \\ (-3 \times 10 + 4 \times 1) \end{bmatrix} = \frac{1}{13}\begin{bmatrix} 13 \\ -26 \end{bmatrix}$$ $$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13/13 \\ -26/13 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$ $$x = 1 \qquad y = -2$$